

2023 CEED MOCK
ADDITIONAL MATHS MARKING KEY
PAPER II

1(a) $P(\text{club}) = \frac{13}{52} = \frac{1}{4}$, there are 13 clubs 1mark

$P(\text{Queen}) = \frac{4}{52} = \frac{1}{13}$ since there are 4 Queens out of 52 cards 1mark

So $P(\text{club \& queen}) = \frac{1}{4} \times \frac{1}{13}$ 2marks

$= \frac{1}{52}$ 1mark

(b)

$\underline{F} + \underline{G} + \underline{H} = (\underline{i} - 2\underline{j}) + (2\underline{i} + \underline{j}) + (\underline{i} + 3\underline{j})$ **1mark**
 $= 4\underline{i} + 2\underline{j}$ **1mark**

From $F = ma$, $a = \frac{F}{m}$ 1mark

$\underline{a} = \frac{1}{10}(4\underline{i} + 2\underline{j})$ **1mark**

$= (\frac{2}{5}\underline{i} + \frac{1}{5}\underline{j}) m/s^2$ **1mark**

2(a) **Area under the graph** $= \frac{1}{2} \times 2 \times t + \frac{1}{2} (20 + t) \times 4$ **1mark, 1mark**
 $= t + 40 + 2t$ **1mark**

But distance covered = 220Km

$t + 40 + 2t = 220$ **1mark**

$3t = 180$ **1mark**

$t = 60$ **1mark**

(b)

Let the position vector of point P be $x\underline{i} + y\underline{j}$

Since Vectors $\overrightarrow{PA}$ and $\overrightarrow{OB}$ are parallel then

$m_{PA} = m_{OB} \rightarrow \frac{4-y}{3-x} = \frac{2}{4} \rightarrow 16 - 4y = 6 - 2x$ **1mark**

$\rightarrow 2y - x = 5 \dots \dots (i)$ **(1mark)**

Also regarding that $\overrightarrow{PA}$ is $\perp$ to $\overrightarrow{PO}$ then

$[(3-x)\underline{i} + (4-y)\underline{j}] \cdot [(-x)\underline{i} - y\underline{j}] = 0$ **1mark**

$-3x + x^2 - 4y + y^2 = 0 \dots \dots \dots (ii)$

from (i) $x = 2y - 5$, then subst. in (ii) gives; (1mark)

$-3(2y - 5) + (2y - 5)^2 - 4y + y^2 = 0$

$-6y + 15 + 4y^2 - 20y + 25 - 4y + y^2 = 0$

$5y^2 - 30y + 40 = 0 \rightarrow y^2 - 6y + 8 = 0$

$$(y - 2)(y - 4) = 0 \rightarrow y = 2 \text{ or } 4$$

true value of y is 2

subst. 2 for y in (i) gives;

$$2(2) - x = 5 \rightarrow x = -1$$

$\therefore$ the position vector of point p is $-\underline{i} + 2\underline{j}$ 1mark

3(a)

$$\overrightarrow{OM} = \overrightarrow{OB} + \overrightarrow{BM} \quad 1\text{mark}$$

$$= \underline{b} + q(\underline{b} + \frac{2}{3}\overrightarrow{OA}) \quad 1\text{mark}$$

$$= \underline{b} + q(\underline{b} + \frac{2}{3}\underline{a}) \quad 1\text{mark}$$

$$= \underline{b} + q\underline{b} + \frac{2}{3}q\underline{a}$$

$$= \underline{b}(1 + q) + \frac{2}{3}q\underline{a} \quad (1\text{marks})$$

(b)

$$\sum fx^2 = 3(10)^2 + 5(15)^2 + 8(20)^2 + 10(25)^2 + 8(30)^2 + 6(35)^2 + 4(40)^2 = 31825$$

1mark mid point, 1 mark multiplying frequency by mid point

$$\sum fx = 3(10) + 5(15) + 8(20) + 10(25) + 8(30) + 6(35) + 4(40) = 1125$$

1mark

$$n = \sum f = 3 + 5 + 8 + 10 + 8 + 6 + 4 = 44 \quad 1\text{mark}$$

$$\text{from } \sigma^2 = \frac{\sum fx^2 - \frac{(\sum fx)^2}{\sum f}}{\sum f}$$

$$\sigma^2 = \frac{31825 - \frac{(1125)^2}{44}}{44} \quad 1\text{mark}$$

$$= \frac{134675}{1936} \quad 1\text{mark}$$

$$= 69.6 \text{ to 1d.p.} \quad (1\text{mark})$$

4 (a)

$$= \sum_{k=1}^7 (k - 2)^2$$

$$= \sum_{k=1}^7 (k^2 - 4k + 4) \quad 1\text{mark}$$

$$= \sum_{k=1}^7 k^2 - 4 \sum_{k=1}^7 k + \sum_{k=1}^7 4 \quad 1\text{ mark}$$

$$= \frac{7(7+1)(2 \cdot 7+1)}{6} - 4 \frac{7(7+1)}{2} + 7 \cdot 4 \quad 1\text{mark}$$

$$= 140 - 112 + 28 \quad 1\text{mark}$$

$$= 56 \text{ Answer} \quad 1\text{mark}$$

(b)

If $\tan \theta = \frac{4}{3}$, then $\sin \theta = \frac{4}{5}$ and $\cos \theta = \frac{3}{5}$ 1mark, 1mark

(a) Regarding vertical motion taking $s=15$

$$\text{From } s = ut + \frac{1}{2}at^2 \quad 1\text{mark}$$

$$\rightarrow 15 = (25\sin\theta)t + \frac{1}{2} \times -10 \times t^2 \quad 1\text{mark}$$

$$15 = \left(25 \times \frac{4}{5}\right)t + \frac{1}{2} \times -10 \times t^2$$

$$\rightarrow 5t^2 - 20t + 15 = 0 \quad 1\text{mark}$$

$$t^2 - 4t + 3 = 0$$

$$\rightarrow (t-1)(t-3) = 0 \quad 1\text{mark}$$

$$\rightarrow t = 1 \text{ or } 3 \quad 1\text{mark, 1mark}$$

$\therefore$ the ball is 15m above the ground 1s and 3s after being struck

5

$$f_x = F\cos 30^\circ \text{ and } f_y = 2F - F\sin 30^\circ \quad 1\text{mark, 1mark}$$

$$\text{Now from } R^2 = (f_x)^2 + (f_y)^2$$

$$(F\cos 30^\circ)^2 + (2F - F\sin 30^\circ)^2 = 12^2 \quad 1\text{mark}$$

$$F^2\cos^2 30^\circ + 4F^2 - 4F^2\sin 30^\circ + F^2\sin^2 30^\circ = 144 \quad 1\text{mark}$$

$$F^2(\sin^2 30^\circ + \cos^2 30^\circ) + 4F^2 - 4F^2\sin 30^\circ = 144 \quad 1\text{mark}$$

$$5F^2 - 4F^2 \times \frac{1}{2} = 144 \quad 1\text{mark}$$

$$\rightarrow 3F^2 = 144 \rightarrow \underline{F = 4\sqrt{3}N}$$

$$f_y = 2F - X - F\sin 30^\circ$$

$$0 = 2 \times 4\sqrt{3} - X - 4\sqrt{3} \times \frac{1}{2} \quad 1\text{mark}$$

$$\rightarrow \underline{X = 6\sqrt{3}N} \quad 1\text{mark}$$

6.

$$\text{Equation of line XY is } r = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + k \begin{pmatrix} 4-1 \\ 3-1 \end{pmatrix} \quad 1\text{mark}$$

$$= \begin{pmatrix} 1 \\ 2 \end{pmatrix} + k \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad 1\text{mark}$$

$$\text{At intersection, } \begin{pmatrix} 1 \\ 2 \end{pmatrix} + k \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + t \begin{pmatrix} 6 \\ 2 \end{pmatrix} \quad 1\text{mark}$$

$$1 + 3k = 4 + 6t \quad 1\text{mark}$$

$$3k - 6t = 3 \text{-----(i)}$$

$$\text{and } 2 + 2k = 1 + 2t \quad 1\text{mark, any one equation}$$

$$2k - 2t = -1 \text{-----(ii)}$$

Solving simultaneously, $t = \frac{-1}{6}$ m1 for solving simultaneous, 1mark for the answer

7 (a)

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 7 + 1 + 1 \quad 1\text{mark, } 1\text{mark}$$

$$(x - 1)^2 + (y - 1)^2 = 9 \quad 1\text{mark, } 1\text{mark}$$

$$(x - 1)^2 + (y - 1)^2 = 3^2 \quad 1\text{mark}$$

The center of the circle is (1,1) 1mark

$$\text{Gradient of the straight line} = \frac{-3}{4} \quad 1\text{mark}$$

$$\text{Gradient of the diameter} = \frac{4}{3} \quad 1\text{mark}$$

$$\text{Eqn. of diameter is } (y - 1) = \frac{4}{3}(x - 1) \quad 1\text{mark}$$

$$y - 1 = \frac{4}{3}x - \frac{4}{3} \quad 1\text{mark}$$

$$3y - 3 = 4x - 4 \quad 1\text{mark}$$

$$3y - 4x = -1 \quad 1\text{mark}$$

(b)

Total number of students = 48

$$P(\text{Add Maths}) = \frac{6}{48} \quad 1\text{mark}$$

$$P(\text{Agriculture}) = \frac{12}{48} \quad 1\text{mark}$$

$$P(\text{Add maths or Agr}) = \frac{6}{48} + \frac{12}{48} - \frac{10}{48} \quad 1\text{mark, } 1\text{mark, } 1\text{mark}$$

$$= \frac{8}{48} \quad 1\text{mark}$$

$$= 0.167 \quad 1\text{mark, } B1 \text{ mark}$$

$$8. (a) \quad \overrightarrow{YZ} = \begin{pmatrix} 4 - 3 \\ 2 - 2 \end{pmatrix} \quad 1\text{mark}$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad 1\text{mark}$$

$$\overrightarrow{XZ} = \begin{pmatrix} 6 - 4 \\ 5 - 2 \end{pmatrix} \quad 1\text{mark}$$

$$= \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad 1\text{mark}$$

$$|\overrightarrow{YZ} + \overrightarrow{XZ}| = \left| \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right| \quad 1\text{mark}$$

$$= \left| \begin{pmatrix} 3 \\ 3 \end{pmatrix} \right| \quad 1\text{mark}$$

$$= \sqrt{3^2 + 3^2} \quad 1\text{mark}$$

$$= \sqrt{27}$$

$$= 5.19 \text{ 1mark}$$

(b)

$$Acc. = \frac{v}{9.6} \text{ m/s/s 1mark, 1mark}$$

$$S_1 = \frac{v^2 - 0^2}{2 \times \frac{v}{9.6}} \text{ 1mark, 1mark}$$

$$= \frac{4.8v^2}{v} \text{ 1mark}$$

$$= 4.8v \text{ 1mark}$$

$$S_2 = \frac{25^2 - 0^2}{2 \times 2.5}$$

$$= \frac{625 - v^2}{5}$$

$$4.8v + \frac{625 - v^2}{5} = 152 \text{ 1mark}$$

$$24v + 625 - v^2 = 760 \text{ 1 mark}$$

$$v^2 - 24v + 135 = 0 \text{ 1mark}$$

$$(v - 15)(v - 9) = 0 \text{ 1mark}$$

$$v = 15 \text{ or } v = 9 \text{ 1mark, 1mark}$$

9. (a)

$$\begin{bmatrix} 1 & -3 \\ 4 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 5 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix} \text{ 1mark}$$

$$Det = [6 \times 2 - 4 \times 2] = 12 - 8 = 4 \text{ 1mark}$$

$$Cofactor \text{ matrix} = \begin{bmatrix} 2 & -4 \\ -2 & 6 \end{bmatrix} \text{ 1mark}$$

$$Transpose \text{ of the Cofactor matrix} = \begin{pmatrix} 2 & -2 \\ -4 & 6 \end{pmatrix} \text{ 1mark}$$

Thus the inverse of $\begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$ is

$$A^{-1} = \frac{1}{det A} \begin{bmatrix} 2 & -2 \\ -4 & 6 \end{bmatrix} \text{ 1mark}$$

$$= \frac{1}{4} \begin{pmatrix} 2 & -2 \\ -4 & 6 \end{pmatrix} \text{ 1mark}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -1 & \frac{3}{2} \end{pmatrix} \text{ 1mark}$$

(b).

Sum of OX-components:

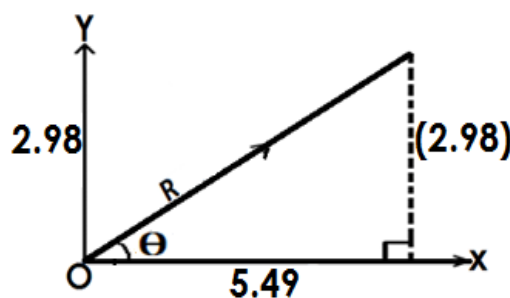
$$= 10\cos 45^\circ + 8\sin 60^\circ - \cos 50^\circ - 7\cos 20^\circ \quad 1\text{mark}$$

$$= 10 \times \frac{1}{\sqrt{2}} + 8 \times \frac{\sqrt{3}}{2} - \cos 50^\circ - 7\cos 20^\circ = 5.49\text{N to 2d.p.} \quad 1\text{mark}$$

Sum of OY-components:

$$= 10\sin 45^\circ + 8\cos 60^\circ - 3\sin 50^\circ - 7\sin 20^\circ \quad 1\text{mark}$$

$$= 10 \times \frac{1}{\sqrt{2}} + 8 \times \frac{1}{2} - 3\sin 50^\circ - 7\sin 20^\circ = 2.98\text{N to 2d.p.} \quad 1\text{mark}$$



B1 mark

$$R^2 = (5.49)^2 + (2.98)^2 \text{ (Pyth theor.)} \quad 1\text{mark}$$

$$R = \sqrt{(5.49)^2 + (2.98)^2} = 6.25\text{N to 2d.p.} \quad 1\text{mark}$$

$$(i) \quad \text{acceleration} = \frac{500\text{kg}}{6.25\text{N}} \quad 1\text{mark}$$

$$= 80\text{m/s/s} \quad 1\text{mark}$$

$$(ii) \text{ Direction} \rightarrow \tan \theta = \frac{2.98}{5.49} \quad 1\text{mark}$$

$$\theta = \tan^{-1} \left(\frac{2.98}{5.49} \right) = 28.49^\circ \text{ to OX} \quad 1\text{mark, 1mark, 1mark}$$

END OF MARKING KEY

END OF QUESTION PAPER

