

2023 CEED MOCK EXAMINATIONS
ADDITIONAL MATHEMATICS MARKING KEY
PAPER I

$$\begin{aligned}
 1. (a) \quad \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1}{h} &= \lim_{h \rightarrow 0} \frac{(h^2 + 2h + 1) - 1}{h} \quad 1\text{mark} \\
 &= \lim_{h \rightarrow 0} \frac{(h^2 + 2h)}{h} \quad 1\text{mark} \\
 &= \lim_{h \rightarrow 0} \frac{h(2+h)}{h} \quad 1\text{mark} \\
 &= \lim_{h \rightarrow 0} h + 2 \quad 1\text{mark} \\
 &= \lim_{h \rightarrow 0} h + \lim_{h \rightarrow 0} 2 \\
 &= 0 + 2 \\
 &= 2 \quad 1\text{mark}
 \end{aligned}$$

1(b)

$$\begin{aligned}
 f(\pi) + g(\pi) &= \cos(\pi) + \pi - \frac{\pi}{2} \quad 1\text{mark} \\
 &= -1 + 1.5708 \\
 &= 0.571
 \end{aligned}$$

2a

$$\begin{aligned}
 (a + bx)^5 &= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \\
 (a + bx)^5 &= a^5 + 5a^4bx + 10a^3b^2x^2 + 10a^2b^3x^3 + 5ab^4x^4 + b^5x^5 \quad 1\text{mark} \\
 \text{But } a^5 + 5a^4bx + 10a^3b^2x^2 &= 32 + 40x + 20x^2 \quad 1\text{mark} \\
 \text{Hence } a^5 &= 32 \\
 a^5 &= 5^2 \\
 a &= 2 \quad 1\text{mark} \\
 \text{Then } 5(2)^4bx &= 40x \quad 1\text{mark} \\
 80b &= 40 \\
 b &= \frac{1}{2} \quad 1\text{mark}
 \end{aligned}$$



$$2b \quad +\left(\frac{2x-1}{3}\right) \geq 2 \quad \text{or} \quad -\left(\frac{2x-1}{3}\right) \geq 2 \quad 1\text{mark}, 1\text{mark}$$

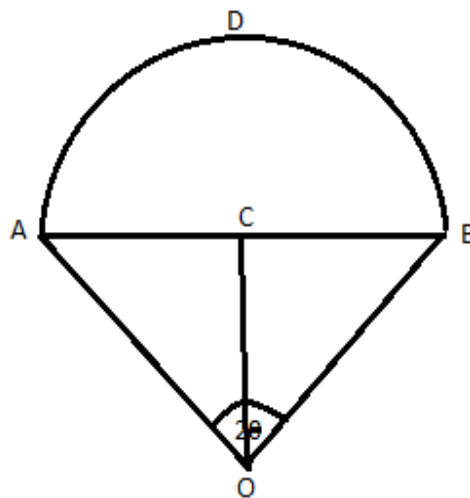
$$2x-1 \geq 6 \quad \text{or} \quad -2x+1 \geq 6 \quad 1\text{mark}$$

$$2x \geq 6+1 \quad \text{or} \quad -2x \geq 6-1 \quad 1\text{mark}$$

$$2x \geq 7 \quad \text{or} \quad -2x \geq 5 \quad 1\text{mark}$$

$$x \geq \frac{7}{2} \quad \text{or} \quad x \geq -\frac{5}{2} \quad 1\text{ mark}$$

3a



Angle AOC = θ

$AC = r \sin \theta$ (SOHCAHTOA) 1mark

Diameter $AB = 2r \sin \theta$ 1mark

But θ is now 2θ 1mark

$\therefore$ Area of a sector OADC = $\frac{1}{2}r^2(2\theta) = r^2\theta$ 1mark

Area of segment ADC = area of sector OADC – area of $\triangle AOC$

$= r^2\theta - \frac{1}{2}r^2 \sin 2\theta$ 1mark

$= r^2(\theta - \frac{1}{2} \sin 2\theta)$ (1mark)

3b

$$\frac{1-\cos 2\theta}{3+\cos 2\theta} = \frac{1-(1-2\sin^2\theta)}{3+(2\cos^2\theta-1)} \quad 1\text{mark}$$

$$= \frac{2\sin^2\theta}{2+2\cos^2\theta} \quad 1\text{mark}$$

$$= \frac{2\sin^2\theta}{2(1+\cos^2\theta)} \quad 1\text{mark}$$

$$= \frac{\cancel{2}\sin^2\theta}{\cancel{2}(1+\cos^2\theta)} \quad 1\text{mark}$$

$$= \frac{\sin^2\theta}{(1+\cos^2\theta)} \quad 1\text{mark}$$

$$\therefore \frac{1-\cos 2\theta}{3+\cos 2\theta} = \frac{\sin^2\theta}{1+\cos^2\theta} \quad 1\text{mark}$$

4a

$$y = \log kn^x$$

$$\log y = \log k + \log x^n$$

$\log y = \log k + n \log x$, thus $n = \text{gradient}$ and $\log k$ is y -intercept. 1mark

$\log x$	2	2.10	2.18	2.24	2.30
$\log y$	1.77	1.63	1.52	1.43	1.35

1mark

Use points G and F or (any other pair) to find gradient,

$$n = \frac{1.52-1.77}{2.18-2}$$

$$= -1.389$$

$$n = -1.4 \text{ to 1d.p.} \quad 1\text{mark}$$

$$1.77 = -1.4(2) + \log k \text{ (or use any pair of numbers from the table for } x \text{ and } y)$$

1mark

$$\log k = 1.76 + 2.8$$

$$= 4.57$$

$$k = 10^{4.5775}$$

$$k = 37153 \quad 1\text{mark}$$

4b

$$V = \pi r^2 h$$

$$\rightarrow h = \frac{V}{\pi r^2} \text{----- (i)} \quad (1\text{mark})$$

$$\text{Surface area (A)} = 2\pi r^2 + 2\pi r h \text{--- (ii)} \quad (1\text{mark})$$

Sub. $\frac{V}{\pi r^2}$ for h in (ii) gives

$$A = 2\pi r^2 + 2\pi r \left(\frac{V}{\pi r^2} \right)$$

$$= 2\pi r^2 + 2 \left(\frac{V}{r} \right) \quad (1\text{mark})$$

$$\frac{dA}{dr} = \left(4\pi r - \frac{2V}{r^2} \right) \quad 1\text{mark}$$



When surface Area is min. $\frac{dA}{dr} = 0$

$$\left(4\pi r - \frac{2V}{r^2}\right) = 0 \quad 1\text{mark}$$

$$\rightarrow 4\pi r^3 - 2V = 0$$

$$r^3 = \frac{2V}{4\pi}$$

$$r = \sqrt[3]{\frac{2V}{4\pi}} \quad 1\text{mark}$$

5

$$\text{From } \sin P + \sin Q = 2 \sin\left(\frac{P+Q}{2}\right) \cos\left(\frac{P-Q}{2}\right) \quad 1\text{mark}$$

$$\begin{aligned} \sin 3x + \sin x &= 2 \sin\left(\frac{3x+x}{2}\right) \cos\left(\frac{3x-x}{2}\right) \\ &= 2 \sin 2x \cos x \quad 1\text{mark} \end{aligned}$$

$$\text{Thus } 2 \sin 2x \cos x = \cos x \quad 1\text{mark}$$

$$2 \sin 2x \cos x - \cos x = 0$$

$$\rightarrow \cos x (2 \sin 2x - 1) = 0 \quad 1\text{mark}$$

$$\text{Either } \cos x = 0 \rightarrow x = 90^\circ, 270^\circ \quad 1\text{mark}$$

$$\text{Or } \sin 2x = \frac{1}{2} \rightarrow 2x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ, 150^\circ, 510^\circ$$

$$x = 15^\circ, 75^\circ, 255^\circ$$

$$\text{As } 0^\circ \leq x \leq 180^\circ, x = 15^\circ, 30^\circ, 75^\circ, 90^\circ \quad 1\text{mark}$$

6.

$$\frac{d}{dx}(x + \cos 2x) = 1 - 2 \sin 2x \quad (1\text{mark})$$

$$\text{At maxima, } 1 - 2 \sin 2x = 0$$

$$\sin 2x = \frac{1}{2}$$

$$2x = \sin^{-1}\left(\frac{1}{2}\right)$$

$$x = 15^\circ \text{ and } 75^\circ \quad (1\text{mark})$$

$$y - \text{values are } \left(\frac{\sqrt{3}}{2} + 15^\circ\right) \text{ and } \left(-\frac{\sqrt{3}}{2} + 75^\circ\right) \quad (1\text{mark})$$

$$\text{i.e. Stat. Points are } \left(15^\circ, \left(\frac{\sqrt{3}}{2} + 15^\circ\right)\right) \& \left(75^\circ, \left(-\frac{\sqrt{3}}{2} + 75^\circ\right)\right) \quad 1\text{mark}$$

$$\frac{d^2y}{dx^2}(x + \cos 2x) = -4 \cos(2x) \quad (1\text{mark})$$

$$\text{At } \left(15^\circ, \left(\frac{\sqrt{3}}{2} + 15^\circ\right)\right), \frac{d^2y}{dx^2} = -4 \cos(2x) = -4 \cos(2(15^\circ)) = -2\sqrt{3} \text{ (max.)}$$

(1mark) for maxima or minima

$$\text{At } \left(75^\circ, \left(-\frac{\sqrt{3}}{2} + 75^\circ\right)\right), \frac{d^2y}{dx^2} = -4 \cos(2(75^\circ)) = -4 \cos(2(75^\circ)) = 2\sqrt{3} \text{ (min.)}$$



$$\frac{d}{dx}(x + \sin 2x) = 1 + 2\cos 2x$$

The curve is max where; $\frac{dy}{dx} = 0$;

$$1 + 2\cos 2x = 0 \rightarrow 2x = \cos^{-1}(-\frac{1}{2})$$

$$2x = \frac{2\pi}{3}, \frac{4\pi}{3} \rightarrow x = \frac{\pi}{3} \text{ and } \frac{2\pi}{3}$$

i.e the true value of x is $\frac{\pi}{3}$ (1mark)

$$\therefore \text{Max. value of } y = \frac{\pi}{3} + \sin 2\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}+2\pi}{6} = 1.9 \text{ to 1d.p. (1mark)}$$

$$\text{Area} = \int_0^{\frac{\pi}{2}} (x + \sin 2x) dx = \left[\frac{x^2}{2} - \frac{\cos 2x}{2} \right]_0^{\frac{\pi}{2}}$$

$$= \left(\frac{\left(\frac{\pi}{2}\right)^2}{2} - \frac{\cos 2\left(\frac{\pi}{2}\right)}{2} \right) - \left(\frac{(0)^2}{2} - \frac{\cos 2(0)}{2} \right)$$

$$= \frac{\pi^2}{8} + 1$$

$$= 2.23 \text{ square units to 2d.p. 1mark}$$

7.

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4 \cos(2\pi - x) dx = 4 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(2\pi - x) dx$$

$$= (4 \sin(x - 2\pi)) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 4 \sin\left(\frac{\pi}{2} - 2\pi\right) - 4 \sin\left(\frac{\pi}{6} - 2\pi\right)$$

$$= 4 \sin \frac{-3\pi}{2} - 4 \sin \frac{-11\pi}{6}$$

$$= -2.51 \text{ approx. (from the calculator)}$$

8(a)

$$\frac{dy}{dx} = 2 - 2x \text{ 1mark}$$

$$\text{At } (2, 0), \frac{dy}{dx} = 2 - 2(2) \text{ 1mark}$$

$$= 2 - 4$$

$$= -2 \text{ 1 mark}$$

$$\text{i.e gradient of tangent } (m_t) = -2 \text{ (1mark)}$$

$$\text{Now eqn is } y - y_1 = m(x - x_1)$$

$$\rightarrow y - 0 = -2(x - 2) \text{ 1mark}$$

$$y = -2x + 4 \text{ 1mark}$$

$$\text{gradient of normal } (m_n) = \frac{-1}{m_t} = \frac{-1}{-2} \text{ 1mark}$$

$$= \frac{1}{2} \text{ (1mark)}$$

$$\text{Now eqn of the normal is } y - y_1 = m(x - x_1)$$

$$\rightarrow y - 0 = \frac{1}{2}(x - 2)$$



$$y = \frac{1}{2}x - 1$$

When $x = 0$, $y = -1$ 1mark

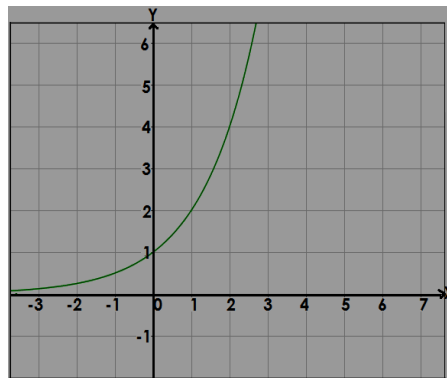
$$P = (0, -1)$$

(1mark)

(b)

x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

1 mark for x values and 1 mark for y values



1mark, plotting points
 1mark, joining with a curve
 1mark, labelling axes
 1mark, correct arrangement of axes
 1mark, correct smooth curve
 1mark, 1mark for showing each line for finding the required value
 From the graph $x = 2.32$ when $y = 5$ (1mark)

9(a) Surface area $= \pi r^2 + ch + 2\pi^2$

$= 3\pi r^2 + 2\pi rh$ (i) 1mark

But Volume of a sphere = $\frac{4}{3}\pi r^3$

$\therefore$ Volume of a hemisphere = $\frac{2}{3}\pi r^3$ 1mark

Hence $\frac{2}{3}\pi r^3 + \pi r^2 h = 45\pi$ 1mark

$$\pi (r^2 h + \frac{2}{3}r^3) = 45\pi$$

$$\therefore r^2 h + \frac{2}{3}r^3 = 45$$

$$h = (45 - \frac{2}{3}r^3)/r^2 \dots\dots\dots(ii) \quad 1mark$$

Substitute (ii) into (i) : Surface area = $3\pi r^2 + 2\pi r((45 - \frac{2}{3}r^3)/r^2)$ 1mark

$$= \frac{3\pi r^4 + 2\pi r(45 - \frac{2}{3}r^3)}{r^2}$$

$$= \frac{3\pi r^4 + 90\pi r - \frac{4}{3}\pi r^4}{r^2}$$

$$= \frac{5\pi r^4 + 270\pi r}{3r^2}$$

$$= \frac{5\pi r^2}{3} + \frac{90\pi}{r} \quad 1mark$$

$$\frac{dA}{dr} = \frac{10}{3}\pi r - \frac{90\pi}{r^2} \quad 1mark$$

At maximum, $\frac{10}{3}\pi r - \frac{90\pi}{r^2} = 0$ 1mark

$$10\pi r^3 - 270\pi = 0$$

$$10\pi r^3 = 270\pi \quad 1mark$$

$$10r^3 = 270$$

$$r^3 = 27$$

$$r = 3 \quad 1mark$$



9(b)

$$y = 4x^3 - 3x^4$$

$$dy/dx = 12x^2 - 12x^3$$

$$\text{At T.P, } 12x^2 - 12x^3 = 0$$

$$12x^2(1 - x) = 0$$

$$12x^2 = 0 \text{ or } 1 - x = 0$$

$$x = 0 \text{ or } x = 1$$

T.P are (0,0) and (1,1)

$$d^2y/dx^2 = 24x - 36x^2$$

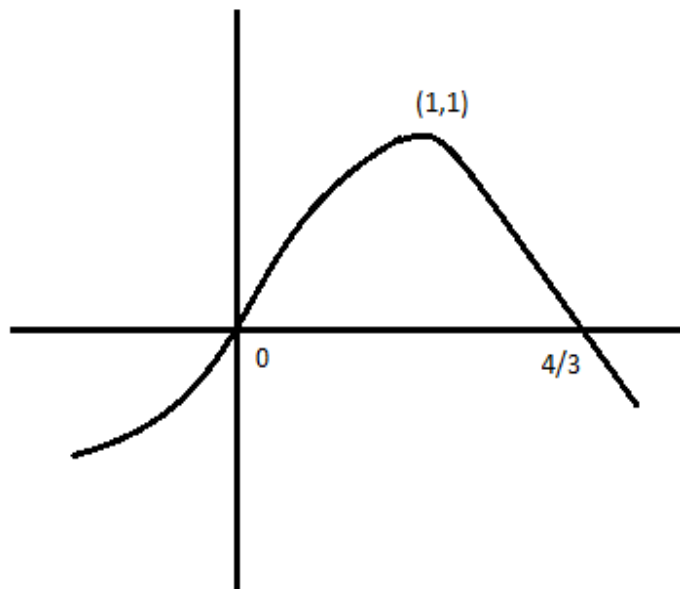
When $x = 0$, $d^2y/dx^2 = 0$ and when x is less than 0, $dy/dx > 0$

(0,0) is a point of inflexion

When $x = 1$ d^2y/dx^2 is greater than -12, so (1,1) is maximum

The graph cuts x axis when $4x^3 - 3x^4 = 0$ that is when $x = 0$ or $4/3$

Sketch



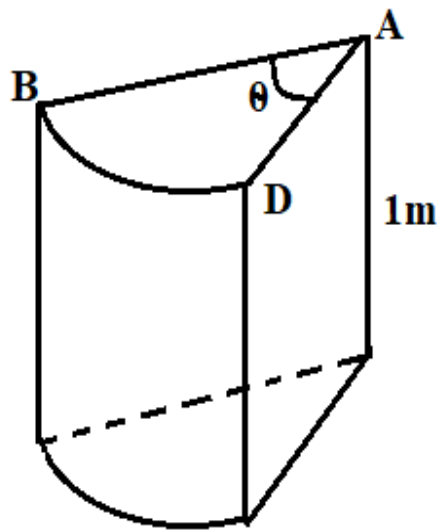
10(a)

$$S = t^2 - kt$$

$$\therefore v = 2t - k \quad 1\text{mark}$$

At inst. rest,	$2t - k = 0$	1mark
	$2(2) - k = 0$	1mark
	$4 - k = 0$	1mark
	$-k = -4$	1mark
	$k = 4$	1mark
	Thus $v = 2t - 4$	1mark
	Hence initial velocity = $2(0) - 4$	1mark
	$= 4\text{m/s}$	1mark 1mark

10(b)



$$\frac{1}{2} r^2 \theta h = 20000 \text{ cm}^3 \quad 1\text{mark}$$

$$r^2 \theta (100\text{cm}) = 40000 \text{ cm}^3$$

$$r^2 \theta = 400 \dots\dots\dots (i) \quad 1\text{mark}$$

Arc length $BD = r\theta$

Hence perimeter = $r + r + r\theta \quad 1\text{mark}$

$$= 2r + r\theta$$

Thus $60 = 2r + r\theta$

$$r = \frac{60}{2 + \theta} \dots\dots\dots (ii) \quad 1\text{mark}$$

Substitute (ii) into (i):

$$\theta \left(\frac{60}{2 + \theta} \right)^2 = 400 \quad 1\text{mark}$$

$$\left(\frac{60\theta}{2 + \theta} \right)^2 = 400$$

$$\frac{3600\theta}{4 + 4\theta + \theta^2} = 400 \quad 1\text{mark}$$

$$3600\theta = 1600 + 1600\theta + 400\theta^2$$

$$4\vartheta^2 + 16\vartheta - 20 = 0 \quad 1\text{mark}$$

$$\vartheta^2 + 4\vartheta - 5 = 0$$

$$(\vartheta - 1)(\vartheta - 5) = 0 \quad 1\text{mark}$$

$$\vartheta = 1 \text{ radian or } \vartheta = 5 \text{ radians} \quad 1\text{mark}, 1\text{mark}$$

END OF MARKING KEY



