**CENTRAL EAST EDUCATION DIVISION**

**2024 MALAWI SCHOOL CERTIFICATE MOCK EXAMINATION**

**PROVISIONAL MARKING KEY**

**ADDITIONAL MATHEMATICS II**

1.a) Using $a=\frac{v-u}{t}$ and $F=m\frac{v-u}{t}$

$F=\sqrt{30^2+40^2}$ ¿ $\sqrt{900+1600}$ ¿ $\sqrt{2500}$ ¿ $50\,N$ (2

$v=\sqrt{12^2+10^2}$ ¿ $\sqrt{144+100}$ ¿ $\sqrt{244}$ (1

So $u=\sqrt{244}-8\left(\frac{5}{50}\right)$ (1

$u=14.8\,m/s$ (1

b)

Expanding $(a-2)^3=a^3-6a^2+12a-8\ +1$ (2

Using Summation Formula

$$\sum_{a=1}^{3}a^3-6\sum_{a-1}^{3}a^2+12\sum_{a=1}^{3}a-\sum_{a=1}^{3}7$$

$$\frac{a^2(a+1)(a+1)}{4}-\frac{a(a+1)(2a+1)}{6}-\frac{a(a+1)}{2}-cn \quad (1$$

$$\frac{3^2(3+1)(3+1)}{4}-\frac{6(3+1)(2(3+1)}{6}-\frac{3(3+1)}{2}-7(3) \quad (1$$

$36-84+72$ (1

$=16$ (1

2. a )

$F_f = \mu F_n$ (1

But $F_n = F_{gy}$=mg cos$\theta$ (1

And $Net\ Force = F_{gx} - F_f$ (1

Therefore $Net\ Force = mg\sin\theta - umg\cos\theta$ (1

$Net\ Force = 6(10)\sin 60° - (0.6)10\cos 60°$ (1

$Net\ Force = 52\,N - 18\,N$ (1

$Net\ Force = 34\,N$ (1

b) Using $s = ut + \frac{1}{2}at^2$

$120 = 2t + \frac{5t^2}{2}$ (1

Now we have $5t^2 + 4t - 240 = 0$ (1

Using quadratic formula

$t = \frac{-t \pm \sqrt{t^2 - 4ac}}{2a}$ (1

t¿ $\frac{-4 \pm \sqrt{4^2 - 4(5)(-240)}}{2(5)}$ (1

t¿ $\frac{-4 \pm \sqrt{16 + 480}}{10}$ (1

$t = 6.54\,s$ or **7 sec** (1

3.a. The numerator is 3 so the summation will have a factor of 3 (1)

$k$will start from zero (1)

There are 5 terms, hence $n=5$ (1)

$The\ sigma\ notation\ is \sum_{k=0}^{5} \frac{3}{5^k}(1)$

b. <u>BA</u> = (-3, 4, 5) (1mark)

Hence $\frac{1}{5}|\overrightarrow{BA}|=\frac{1}{5}\left(\sqrt{(-3)^2+4^2+5^2}\right)(1)$

¿ $\frac{1}{5}\left(\sqrt{9+16+25}\right)(1)$

¿ $\frac{1}{5}\left(\sqrt{50}\right)(1)$

¿ $\frac{1}{5}\left(5\sqrt{2}\right)(1)$

¿ $\sqrt{2}\,units\,(1)$

4 $\overrightarrow{OP}.\overrightarrow{OQ}=\begin{pmatrix}1+\mu\\3-2\mu\\4+2\mu\end{pmatrix}.\begin{pmatrix}4\\2\\3\end{pmatrix}$

$\overrightarrow{OP}.\overrightarrow{OQ}=4(1+\mu)+2(3-2\mu)+3(4+2\mu)(1)$

¿ $4+4\mu+6-4\mu+12+6\mu(1)$

¿ $6\mu+22(1)$

¿ $(6\mu+12)+10(1)$

¿ $2(3\mu+6)+10(1)$

5) $r_1=r_2$

So $\begin{pmatrix}0\\2\end{pmatrix}-t\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}3\\0\end{pmatrix}-u\begin{pmatrix}-6\\3\end{pmatrix}$ (1

$0-3t=3+6u........i$ (1

$2-2t=0-3u.......ii$

Solving the equations simultaneously (1

$t=\frac{1}{7}\wedge u=\frac{-4}{7}$ (1,1

Now substitute the values into the equations (1

$\underline{r}_1=\begin{pmatrix}0\\2\end{pmatrix}+\frac{1}{7}\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}0\\2\end{pmatrix}+\begin{pmatrix}\frac{3}{7}\\\frac{2}{7}\end{pmatrix}$ (1,1

The intersection is $\left(\frac{3}{7},\frac{16}{7}\right)$ (1

6.a. $x=\frac{g}{-2}\,y=\frac{h}{-2}$ (1)

$x=\frac{-4}{-2}=2, y=\frac{2}{-2}=-1$ (1)

Centre = (2, -1)

$r=\sqrt{a^2+b^2-c}$

$r=\sqrt{2^2+(-1)^2-(-8)}(1)$

r ¿ $\sqrt{4+1+8}(1)$

¿ $\sqrt{13}(1)$

b. $let\, a=3\underline{i}+\underline{j}\wedge b=k\underline{i}-3\underline{j}$

$a.b=|a||b|cos\beta(1)$

$\cos\beta=\frac{a.b}{|a||b|}$

$a.b=3k-3(1)$

$a=\sqrt{3^2+1^2}=\sqrt{10}(1)$

$b=\sqrt{k^2+(-3)^2}=\sqrt{k^2+9}(1)$

Hence $\frac{3}{\sqrt{130}}=\frac{3k-3}{\sqrt{10}.\sqrt{k^2+9}}(1)$

$\frac{9}{130}=\frac{(3k-3)^2}{10(k^2+9)}(1)$

$\frac{9}{13}=\frac{9k^2-18k+9}{k^2+9}$

$k^2+9=13k^2-26k+13\Rightarrow 0=12k^2-26k+4(1)$

$0=6k^2-13k+2$

$\therefore k=2\vee\frac{1}{6}(1,1)$

## Section B

7. a) $x+3y=5$

$$-4x+2y=9 \quad (1)$$

$$\begin{pmatrix} 1 & 3 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 5 \\ 9 \end{pmatrix}(1)$$

$$x=\frac{\begin{vmatrix} 5 & 3 \\ 9 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix}}=\frac{10-27}{2+12}=\frac{-17}{14}(3)$$

$$y=\frac{\begin{vmatrix} 1 & 5 \\ -4 & 9 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix}}=\frac{9+20}{2+12}=\frac{29}{14}(3)$$

$$\therefore x=\frac{-17}{14} \wedge y=\frac{29}{14}(1)$$

b.)

$Net\ Force=ma=F_{gx}=mg\sin\theta$ (3

Substituting into the formula, we have (1

$ma=mg\sin\theta$ (1

$8a=8(10)\sin 45°$ (1, 1

$a=\frac{8(10)\sin 45°}{8}$ (1,1

$a=\frac{56.568542495}{8}$ (1

$a=7.0710$ or simply **7m/s/s** (1

8. a)

Midpoint of AB = ½ (2 + 4, 6+8) (1

= (3,7) (1

Gradient of AB = $\frac{6-8}{2-4}$ = 1 (1

$\therefore$Gradient of the perpendicular bisector of AB = $-¿1$ (1

$\therefore$ Equation of perpendicular bisector of AB is y – 7 = $-¿1$(x – 3) (1

$y = -¿x + 10$……….(i)   (1

Midpoint of BC= ½(4+8, 8+10)

= (6,9)

Gradient of AB = $\frac{8-10}{4-8}$ = ½

∴Gradient of the perpendicular bisector of AB = $-¿2$

∴ Equation of perpendicular bisector of AB is y – 9 = $-¿2$(x – 6)

$y = -2x + 21$………(ii)

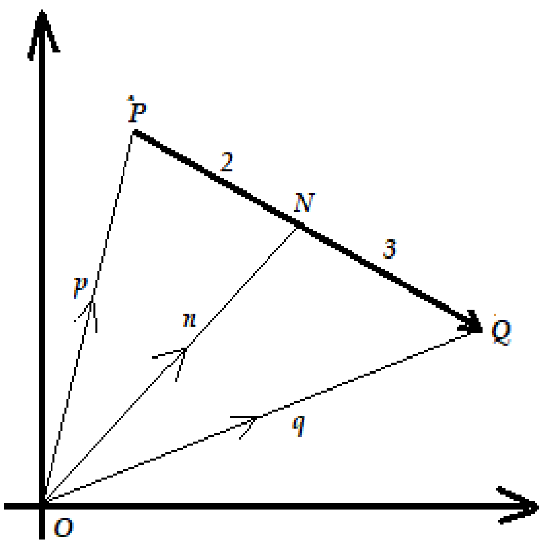
∴Solving (i) and (ii) simultaneously, $y = -¿\,1$ , x = 11   (1, 1, 1

∴Centre of the circle is (11, -1), and using point A or any one of the two other points through which the circle passes, r = $\sqrt{(11-2)^2+(-1-6)^2}$  ( 1 for centre

$= \sqrt{130}$  (1

∴The equation of the circle is $(x-11)^2 + (y+1)^2 = (\sqrt{130})^2$   (1

b) ***On next page***




(1

$\underline{ON} = \underline{OP} + \underline{PN}$

$= \underline{OP} + \frac{2}{5}\underline{PQ}$   (1

$= \underline{OP} + \frac{2}{5}(\underline{PO} + \underline{OQ})$  (1

$= \underline{p} + \frac{2}{5}(-¿\underline{p} + \underline{q})$

$= \underset{\sim}{p} - \frac{2}{5}\underset{\sim}{p} + \frac{2}{5}\underset{\sim}{q}$ (1

$= \frac{3}{5}\underset{\sim}{p} + \frac{2}{5}\underset{\sim}{q}$ (1

The position vector of $\underset{\sim}{p} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} -8 \\ 10 \end{pmatrix}$ (1

$\underline{ON} = \frac{3}{5}\begin{pmatrix} 2 \\ 6 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} -8 \\ 10 \end{pmatrix}$

$= \begin{pmatrix} 1.2 \\ 3.6 \end{pmatrix} + \begin{pmatrix} -3.2 \\ 4 \end{pmatrix}$ (1

$= \begin{pmatrix} -2 \\ 7.6 \end{pmatrix}$ (1

The coordinates of N are (−¿2 , 7.6) (1

9.a. $(x-3)^2 + y^2 - 2x + 2(x-3) + 1 = 0(1)$

$$x^2 - 6x + 9 + x^2 - 2x + 2x - 6 + 1 = 0(1)$$
$$2x^2 - 6x + 4 = 0(1)$$
$$(2x-2)(x-2) = 0(1)$$
$$\therefore x = 1 \vee 2(1)$$

Points of intersection

$$x=1, y=1-3=-2(1)$$
$$x=2, y=2-3=-1(1)$$
$$(1,-2) \wedge (2,-1)(1)$$

The centre of the circle is $(1,-1)$ (1)

Tangent is perpendicular to line from centre.

Line from centre $(1,-1)$ to (1, -2)

$gradient = \frac{-2--1}{1-1} = \infty$ hence the line perpendicular to this will have gradient = 0 (1)

Hence the tangent at (1, -2) is $y=-2$ (1)

Similarly at point (2, -1) the tangent is $x=2$

The two tangents meet at $(2,-2)$ (1)

$\therefore distance ¿ centre ¿ this\ point = \sqrt{(x_1 - x)^2 + (y_1 - y)^2}$ (1)

$$¿\sqrt{(2-1)^2 + (-2--1)^2} = \sqrt{2}$$

$(1)$

b. re-arranging the terms

2, 3, 4, 4, 7, 8, 9, 11, 19, 23, 25, 27. (1)

Lower quartile $¿\frac{n+1}{4}th$ term$¿\frac{13+1}{4}=3.5=4\,th\,term=4$ (1, 1)

Upper quartile $¿\frac{3(n+1)}{4}th$ term $¿\frac{3(13+1)}{4}=11\,th\,term=23$ (1.1)

Interquartile range

$¿upper\ quartile-lower\ quartile=23-4=19(2)$